LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

U.G. DEGREE EXAMINATION – **ALLIED**

FOURTH SEMESTER – **APRIL 2023**

UMT 4402 – MATHEMATICS FOR PHYSICS - II

Date: 04-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

	SECTION A - K1 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
1.	**Answer the following**
a)	What is the Euler's formula to find a_0 in the Fourier series for the function $f(x)$ in the interval $\alpha < x < \alpha + 2\pi$?
b)	What is an ordinary differential equation?
c)	What do you mean by a linear differential equation?
d)	Define Laplace transform.
e)	Define the gradient of a scalar point function f.
2.	**Fill in the blanks**
a)	If $n \neq 0$, then $\int_0^{2\pi} \cos nx \; dx =$ ____________ .
b)	A solution of a differential equation obtained by giving particular values to the arbitrary constants in the complete solution is called as a ______________ .
c)	The sum of the complementary function and the particular integral of an ordinary differential equation is called ____________________________.
d)	$L\{e^{3t}\} =$ ________________.
e)	$div\ curl\ \boldsymbol{F} =$ _____________.
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**Answer the following MCQ**
a)	Which of the following function f is an odd function? i. $f(x) = \cos x$ ii. $f(x) = e^x$ iii. $f(x) = \sin x$ iv. $f(x) = x^2 + x$
b)	What is the integrating factor of the Leibnitz linear equation $\frac{dy}{dx} + Py = Q$? i. e^{PQ} ii. $e^{\int Pdx}$ iii. $e^{\int Qdx}$ iv. $\int Pdx$
c)	The complete solution of $(D^2 - 1)y = 0$ is i. $y = Ae^x + Be^x$ ii. $y = Axe^x + Be^{-x}$ iii. $y = Axe^x + Be^x$ iv. $y = Ae^x + Be^{-x}$
d)	The inverse Laplace transform of $\frac{1}{s^2}$ is i. $2t$ ii. 1 iii. t iv. $\sin t$

e)	If $\boldsymbol{R} = x\boldsymbol{I} + y\boldsymbol{J} + z\boldsymbol{K}$, then $\nabla \cdot \boldsymbol{R}$ is equal to i. 3 ii. 0 iii. -3 iv. 4
4.	**State True or False**
a)	The Fourier series cannot be obtained for a periodic function.
b)	The order of the differential equation $y\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^2 + x$ is 1.
c)	$\frac{1}{D}X = \int X\,dx$.
d)	If $L\{f(t)\} = \bar{f}(s)$, then $L\{e^{ax}f(t)\} = \bar{f}(s+a)$.
e)	The divergence of vector valued function is a scalar point function.

SECTION B - K3 (CO2)

	Answer any TWO of the following in 100 words **(2 x 10 = 20)**
5.	Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.
6.	Solve $y\sqrt{1-x^2}\,dy + x\sqrt{1-y^2}\,dx = 0$.
7.	Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = (1-e^x)^2$.
8.	Draw the graph of the periodic function $f(t) = \begin{cases} t, & when\ 0 < t < \pi \\ \pi - t, & when\ \pi < t < 2\pi \end{cases}$ and find its Laplace transform.

SECTION C – K4 (CO3)

	Answer any TWO of the following in 100 words **(2 x 10 = 20)**
9.	Solve $\left(y^2e^{xy^2} + 4x^3\right)dx + \left(2xye^{xy^2} - 3y^2\right)dy = 0$.
10.	By using the method of variation of parameters solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$.
11.	Find $L^{-1}\left\{\frac{2s^2 - 6s+5}{s^3-6s^2+11s-6}\right\}$.
12.	If $\boldsymbol{F} = 3xy\boldsymbol{I} - y^2\boldsymbol{J}$, evaluate $\int \boldsymbol{F} \cdot d\boldsymbol{R}$ over the curve $y = 2x^2$ in the xy-plane from $(0,0)$ to $(1,2)$.

SECTION D – K5 (CO4)

	Answer any ONE of the following in 250 words **(1 x 20 = 20)**
13.	Prove that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n\frac{\cos nx}{n^2}$, in $-\pi < x < \pi$. Hence show that a) $\sum\frac{1}{n^2} = \frac{\pi^2}{6}$. b) $\sum\frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$. c) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots = \frac{\pi^2}{12}$.
14.	Solve $\frac{dy}{dx} = \frac{y+x-2}{y-x-4}$.

SECTION E – K6 (CO5)

	Answer any ONE of the following in 250 words **(1 x 20 = 20)**
15.	By using Laplace transform solve the differential equation $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$ with $x = 2$ and $\frac{dx}{dt} = -1$ at $t = 0$.

16.	State Gauss divergence theorem and verify the theorem for $\boldsymbol{F} = (x^2 - yz)\boldsymbol{I} + (y^2 - zx)\boldsymbol{J} + (z^2 - xy)\boldsymbol{K}$ taken over the rectangular parallelepiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

$$$$$$$